RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2018

SECOND YEAR (BATCH 2016-19)

Date : 28/05/2018 **MATHEMATICS (General)**

Time : 11.00 am – 2.00 pm Paper : IV Full Marks : 75

[Use a separate Answer Book for each group]

Group - A

Answer <u>any four</u> questions from <u>Question No. 1 to 6</u> : [4×5]

1. Show that $\int_{-1}^{1} \frac{dx}{x^3}$ exists in the Cauchy principal value sense but not in the general sense. [5]

2. a) Show that $\int_0^{\pi/2} \cos^4 x \sin^2 x \, dx = \frac{\pi}{32}$. [3]

   b) Prove that the integral $\int_0^1 \frac{\log x}{\sqrt{x}} dx$ converges. [2]

3. a) Evaluate $\iint_R \sin(x+y) dxdy$ over $R: \left\{0 \le x \le \frac{\pi}{2};\ 0 \le y \le \frac{\pi}{2}\right\}$. [3]

   b) Compute the value of $\iint_R ydxdy$ where R is the region in the first quadrant bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [2]

4. Find the total length of the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$. [5]

5. Prove that the area included between the curves $y^2 = 2ax - x^2$ and $y^2 = ax$ above the x-axis is $\left(\frac{\pi}{4} - \frac{2}{3}\right)a^2$. [5]

6. Find the volume of the solid generated by the revolution of the catenary $y = a\cosh\left(\frac{x}{a}\right)$ between x = c and x = b about the x-axis. [5]

Answer <u>any two</u> questions from <u>Question No. 7 to 9</u> : [2×5]

7. Find the orthogonal trajectories of the family of curves $x^{\frac{4}{3}} + y^{\frac{4}{3}} = a^{\frac{4}{3}}$, where 'a' is a variable parameter. [5]

8. Solve : $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$. [5]

9. Solve : $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 9y = 40\sin 5x$. [5]

Group - B

Answer <u>any three</u> questions from <u>Question No. 10 to 15</u> : [3×15]

10. a) Give the brief idea of primary data and secondary data. Also discuss about the methods of data collection. [4+4]

b) A sample of 280 undergraduate students of Vidyamandira was asked to give their opinion regarding the chance of Kolkata's win in the IPL 2018 prior to the game. Each student was to respond either to 'very high' or 'very poor' or 'cannot comment' on the issue. The following data were obtained.

Responses	**Number of students**
very high	152
very poor	51
cannot comment	77

Draw a pie chart for the obtained data and give your opinion on the basis of the pie chart. [5+2]

11. a) Prepare consumer price index number from the following data for 2010 and 2011 taking 2009 as base year : [6]

Group	Price in Rupees		
	2009	2010	2011
A	20·00	24·00	21·00
B	1·25	1·50	1·00
C	5·00	8·00	8·00
D	2·00	2·25	2·12

It is given that weights of the four groups are 4, 3, 2 and 1 respectively.

b) Fit a straight line trend equation by the method of least squares and estimate the corresponding trend values. [6+3]

Year	1991	1992	1993	1994	1995	1996	1997	1998
values	80	90	92	83	94	99	92	104

12. a) Find out Standard Deviation from the following table giving the age distribution of 540 members of a parliament [5]

Age in years	30	40	50	60	70
Number of members	64	132	153	140	51

b) The joint probability distribution of 2 discrete random variables is [2+3]

$$f(x,y)=\begin{cases}\frac{1}{4}, & x=1, y=0\\ \frac{1}{4}, & x=2, y=3\\ \frac{1}{2}, & x=3, y=5\\ 0, & \text{elsewhere}\end{cases}$$

Obtain (a) the marginal distribution of Y. (b) cov (X,Y) .

c) If $x+2y=5$ and $2x+3y=8$ represent two regression lines, then find the means of x and y. Also find correlation coefficient between x and y. [2+3]

13. a) Define parameter and statistic. Explain why standard deviation of a statistic is called 'standard error'. [2+3]

b) Write down the probability density function of χ^2 distribution with n degrees of freedom. Mention its mean and variance. [2+1]

c) If Y_1, Y_2 are two independent χ^2 variates with n_1 and n_2 degrees of freedom respectively, then what is the distribution of $Y_1 + Y_2$? [2]

d) Given the probability density function

$$f(x) = \begin{cases} \frac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{elsewhere} \end{cases}$$

Suppose we want to test the null hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H : \theta = 2$ on the basis of a single observation x. Find the probabilities of type-I error and type-II error if the critical region be $x > 0{\cdot}5$. [5]

14. a) Show that the probability that exactly one of the events A and B occurs is $P(A) + P(B) - 2P(AB)$. [3]

b) A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from the bag B. [4]

c) The probability that Ashok can solve a problem in Statistics is $\frac{4}{5}$, that Amal can solve it is $\frac{2}{3}$ and that Abdul can solve it is $\frac{3}{7}$. If all of them try to solve independently, find the probability that the problem will be solved. [4]

d) The probabilities that a particular sum will be solved by A and B are $0{\cdot}5$ and $0{\cdot}6$. Find the probability that the sum will be solved. [4]

15. a) Find the mean and variance of the Binomial distribution. [2+3]

b) The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} Kx^2 e^{-\frac{x}{2}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

where K is a real constant. Find (i) the value of K, (ii) $P(X > 2)$. [3+2]

c) The spectrum of random variable X consists of only non-negative integers with probability $P(X = i) = \frac{1}{1+\mu}\left(\frac{\mu}{1+\mu}\right)^i$, $\mu > 0$, $i = 0, 1, 2, 3 \ldots$ Find mean and variance. [2+3]

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